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understood that h depends on both x and e .) From the Heine-Borel theorem (see, for instance, Veblen and Lennes, *Infinitesimal Analysis*, p. 34) it follows that there exists for every e a finite set of points

$$a = x_1 < x_2 < x_3 \cdots, x_{m-1} < x_m = b$$

such that every point of the interval (a, b) belongs to some segment S_{x_i} , and that for every i ($= 1, \cdots, m$) the oscillation of $\delta(x)$ on S_{x_i} is, if we omit the point x_i , less than e . Hence if at any point x the measure of discontinuity of the function $\delta(x)$, and therefore of $f(x)$, exceeds e , x is some one of the points x_i . Assign to e successively the values $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \cdots, \frac{1}{n}, \cdots$ and denote the points corresponding to $e = 1/n$ by x_{ni} ($i = 1, 2, 3, \cdots, m_n$). The set of all points x_{ni} is enumerable and contains the set of all points at which the saltus of $\delta(x)$ is positive, which must therefore be an enumerable set. The function $\delta(x)$, and consequently the function $f(x)$, is continuous for every point x not in the set of points x_{ni} , since such a point belongs (for every value of n) to a segment $S_{x_{ni}}$ on which the oscillation of $\delta(x)$ is less than $1/n$.

Consider the classical example of a function continuous at the irrational points of an interval and discontinuous at every rational point. The function $f(x) = 0$, if x is an irrational point of the interval $(0, 1)$, $f(x) = 1/q$ if $x = p/q$ (p and q relatively prime integers and $p < q$). The function $f'(x)$ exists and vanishes identically. Hence $\delta(x) = -f(x)$. $f(x)$ is discontinuous on a dense enumerable set and possesses the maximum degree of discontinuity permissible under the theorem.

The argument of this note can be extended immediately by means of suitable changes of the terminology so as to apply to any abstract set admitting a definition of distance and the generalized Heine-Borel theorem.

THE NINE-POINT CIRCLE OBTAINED BY METHODS OF PROJECTIVE GEOMETRY.

By H. N. WRIGHT, Whittier College.

Place a mirror of zero dimensions, but with a fixed direction d at a point A . Then any line a through A reflects into a line a' through the same point, and a'

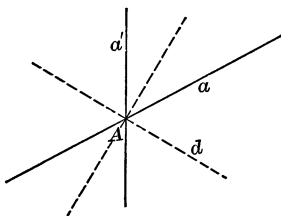


FIG. 1.

reflects back into a . Moreover by considering the angles of reflection it is clear that the pencil described by a is projective to the pencil described by a' . Thus

a and a' are a pair in an involution of lines about A , which we may call a *mirror involution*. The double elements are the line in the direction of the mirror and the line at right angles to it.

Two mirror involutions A and B may be used to set up a quadratic transformation¹ of the plane. Any point P by reflection from A and B goes into a point P' and conversely. In general a straight line transforms into a conic through

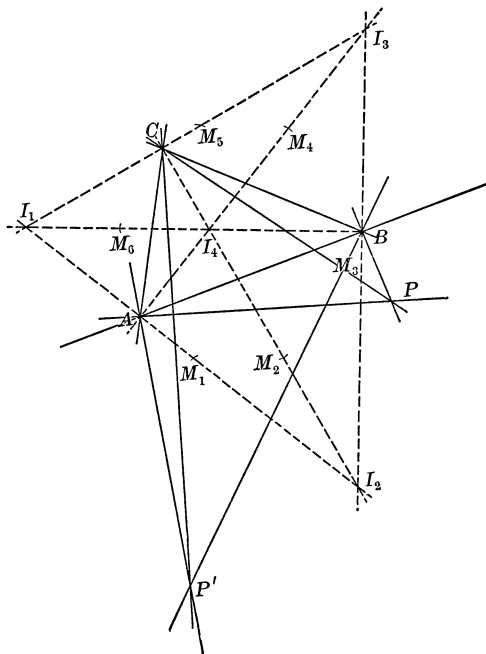


FIG. 2.

A and B . However all points of the line AB , other than the points A and B , go into the same point C . Now if a mirror involution is placed at C with the direction of its mirror bisecting the exterior angle of the triangle ABC at that point, it may be shown that any two of the three involutions yield the same quadratic transformation.² Then a conic which is obtained by transforming a line passes through A , B and C .

By following the construction of the transformation it is seen that the vertices and the ortho-center of the triangle, whose sides are the directions of the mirrors, are the four invariant points I_1 , I_2 , I_3 and I_4 . Also we see that a point of any one of the lines $I_i I_j$ transforms into another point of the same line; thus setting up an involution of points on $I_i I_j$ in which I_i and I_j are the self-corresponding

¹ This is a special case of the quadratic transformation discussed in a paper, "On the Combination of Involutions," by D. N. Lehmer in this MONTHLY for March, 1911.

² See proof in the general case in paper mentioned. We note that this proves the proposition that if three lines, one through each of the vertices of a triangle, are concurrent, then their isogonal conjugates are concurrent.

elements. Hence the midpoint M_k of the inner segment $I_i I_j$ corresponds to the ideal point of the line $I_i I_j$. From this it follows that *the ideal line of the plane transforms into a conic through each of the midpoints of the six segments $I_i I_j$ in addition to passing through A , B and C , as noted above.*

We may now show that *this conic is a circle*. If P is an ideal point the lines a and b joining it to two of the mirrors are parallel. Then the sum of the angles α , β and γ is four right angles. Now as P moves on the ideal line a change in

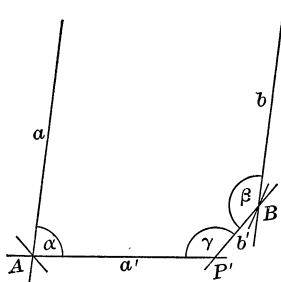


FIG. 3.

the angle α is accompanied by a change in the angle β of the same magnitude but opposite in sign. Then $\alpha + \beta$ is constant and hence $\gamma = 360^\circ - (\alpha + \beta)$ is constant and P' moves on a circle. It follows, then, that *the conic obtained by transforming the ideal line of the plane is the nine-point circle of each of the four triangles $I_i I_j I_k$.*

THE ROCKY MOUNTAIN SECTION.

The second annual meeting of the Rocky Mountain section of the Mathematical Association of America was held at Laramie, Wyoming, under the auspices of the University of Wyoming, March 29 and 30, 1918.

The meeting opened with a dinner in Hoit Hall at 6 P. M., at which the address of welcome by Acting President Nelson of the University of Wyoming and the response by Professor O. C. Lester, of the University of Colorado, were given. After the dinner, an adjournment to the administration building was made and the following program was carried out:

1. Special Courses in Mathematics for Technical Students. PROFESSOR S. L. MACDONALD, Colorado A. & M. College, Ft. Collins.
2. The Theory of the Mercury Arc. PROFESSOR J. W. WOODROW, University of Colorado, Boulder.
3. The Length Integral in the non-Euclidean World of Poincaré. PROFESSOR C. E. STROMQUIST, University of Wyoming, Laramie.

The purpose of this paper was to derive an integral for length in the non-Euclidean world proposed by Poincaré in his "Foundations of Science," English translation by Halsted, page 75. The shortest distance between two points is